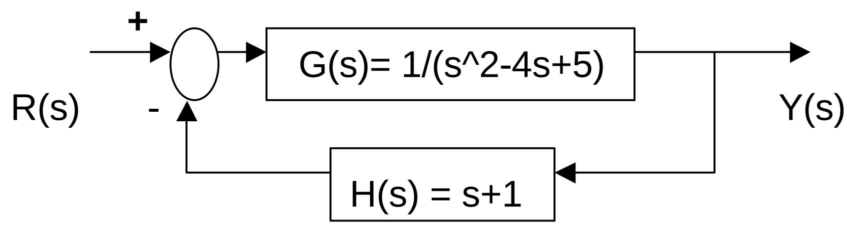
(Sample):

Use Nyquist Criterion and check for system Stability.



Solution:

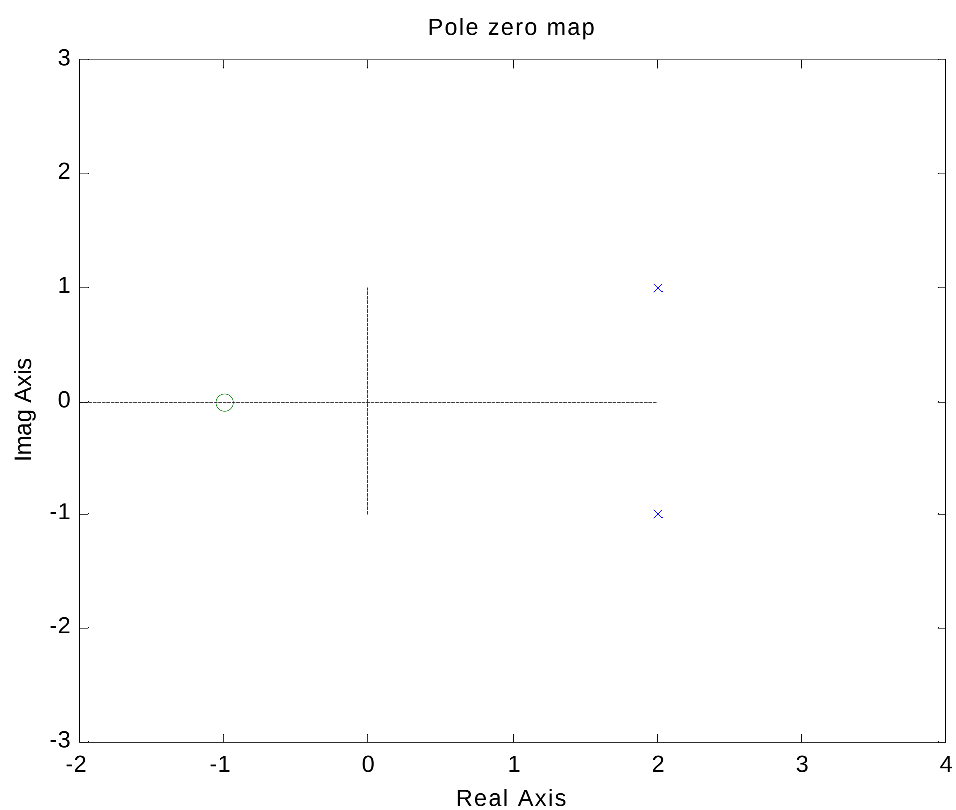
(version 1- by Matlab)

Open Loop TF,
G(s)H(s) = (s+1)/ (s^2-4s+5) = (s+1)/ { (s-2+j) (s-2-j) }

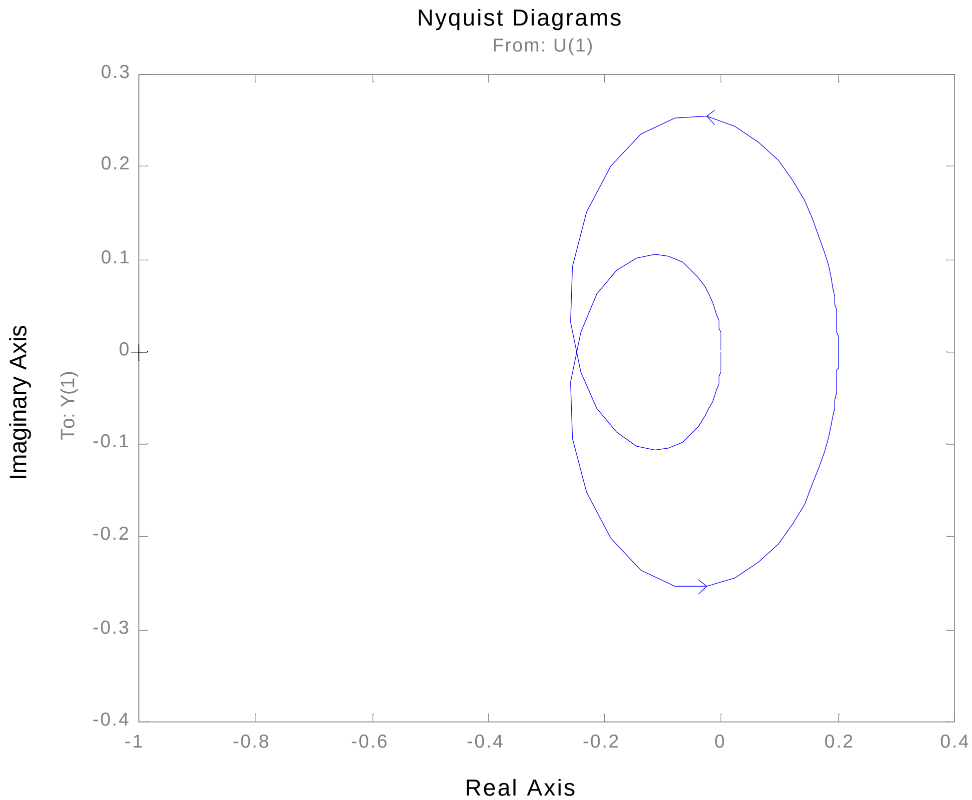
Use contour to represent the RHP in s-plane, (given the PZ map)
In Matlab,

```
>> num = [1,1]; den = [1,-4,5];
>> pzmap(num,den)
```

← edit the Axes as shown below,



>> nyquist(num,den)



Thus,

From G(s)H(s) : #poles on RHP is, P =2,
From Nyquist Plot: #times (-1,0) is encircled is N = 0,

Therefore,

#poles of T(s) on RHP = #zeros of 1+GH(s)=0 on RHP = N+P=2,

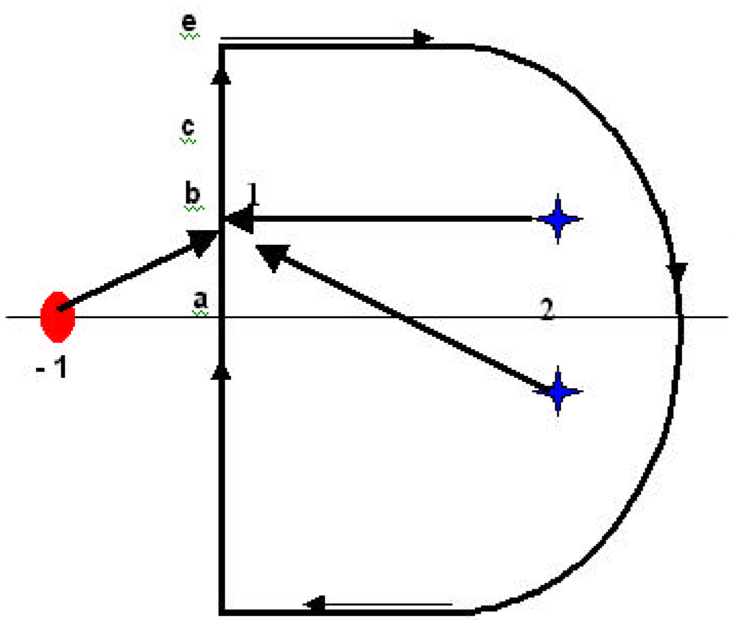
As a consequence, SYSTEM IS UNSTABLE.

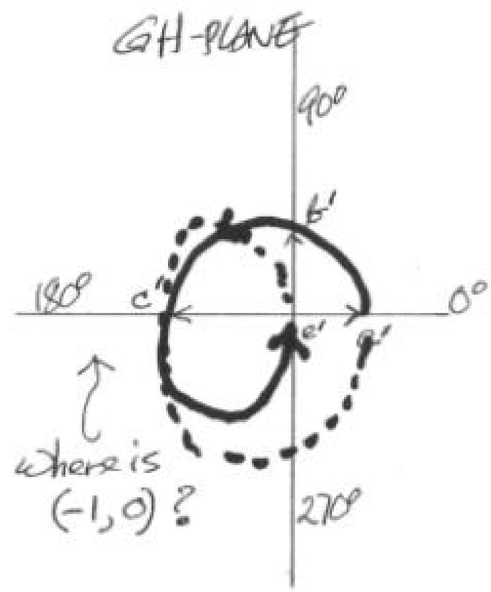
(… continue to next page)

(version 2 – by HAND)

Select s-plane contour to enclosed RHP as shown below, consider four crucial points on jw-axis,

a: $s=j0^+$
b: $s=j$
e: $s=j$ infinity
c: $s=j$ w, where w=?, but angle of c' image in GH = 180^0





<u>At a: the a' image has,</u>

Magnitude = 1/(sqrt(5)*sqrt(5))=1/5
Angle = [0] – [180^0 + arctan(1/2) + 180^0 – arctan (1/2)]=0

<u>At b: the b' image has,</u>

Magnitude = sqrt(2)/[(2)(2*sqrt(2)] = ¼
Angle = [arctan(1)] – [180^0 + 180^0 – arctan(2/2)]= -270^0 = 90^0

<u>At e: the e' image has,</u>

Magnitude = infinity / [infinity * infinity] = 0 (by L'Hopital Rule)
Angle = [90^0] – [90^0 + 90^0] = -90^0 = 270^0

Now to find whether point (-1,0) will be enclosed or not by the completed closed Nyquist Plot, we need to determine the location of point: c on the jw-axis. In short what is the frequency w when at point c.

We use s=jw,

At c: the c' image has,

Magnitude = sqrt(1+w^2) / [(sqrt(4+(w-1)^2)*(sqrt(4+(w+1)^2)]

= ? ← **the value we like to know.**

Angle = [arctan(w)] – [180^0-arctan((w-1)/2)+180^0-arctan((w+1)/2]

= 180^0

Thus,

arctan(w) + arctan((w-1)/2) + arctan((w+1)/2) = 180^0

```
          \               \                        \
          A               B                        C
```

Denote the angles as indicated by A, B and C,

Therefore,

$$A + B + C = 180^0$$

Consider, B + C,

and apply tangent formula, we have,

$$\tan(B + C) = \frac{\tan(B) + \tan(C)}{1 - \tan(B)*\tan(C)} = \frac{(w-1)/2 + (w+1)/2}{1 - (w-1)(w+1)/4}$$

$$= \frac{4w}{5 - w^2}$$

now denote,

D=B+C,

So,

$$A + D = 180^0$$

and,

$$\tan(A+D) = \frac{\tan(A) + \tan(D)}{1 - \tan(A)\tan(D)} = \tan(180^0) = 0$$

thus,

tan(A) + tan(D) = 0

w + 4w / (5-w^2) = 0

9w – w^3 = 0

w (9 – w^2) = 0

since, w is not zero at point c, therefore,

w^2 = 9

and,

w = j3 at point c, that will have 180^0 at image c'.

Finally, the magnitude at c' is,

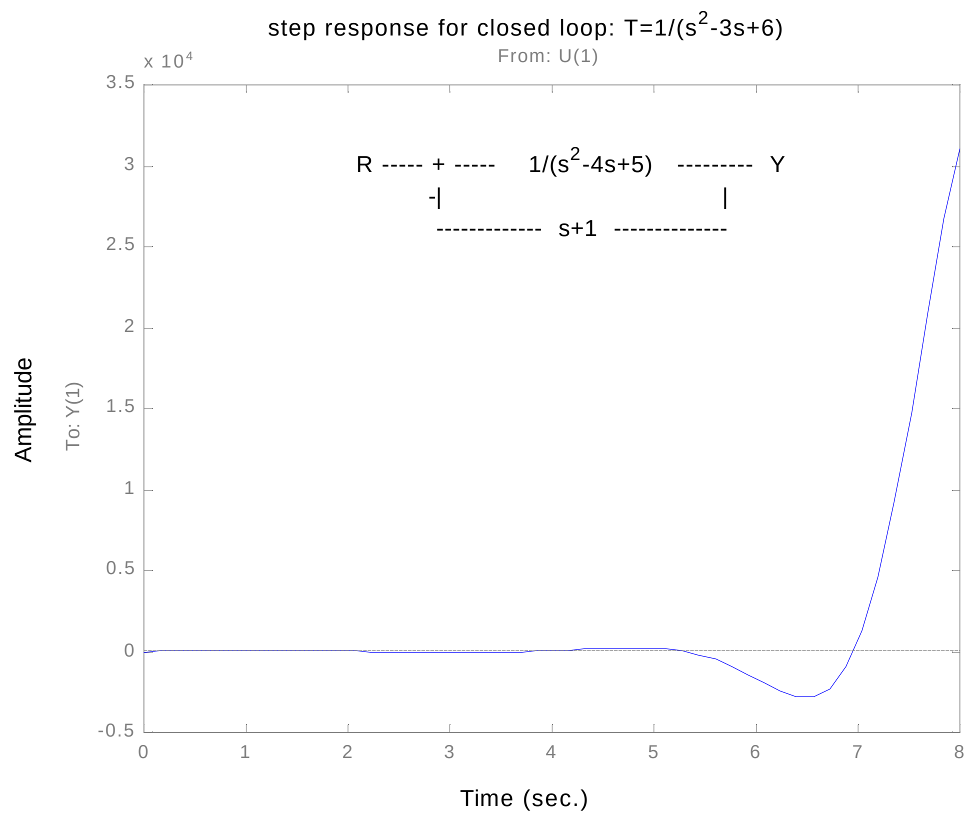
Magnitude = sqrt(1+3^2) / [sqrt(4+2^2)* sqrt(4+4^2)]= ¼

Clearly, point (-1,0) is never encircled in GH-plane. Thus, N=0 and since P=2, the number of zeros of the system closed loop TF = Z = N+P=2, system is **UNSTABLE.**

Checking,

(1) by Matlab,

```
>>num2=1; den2=[1,-3,6]; step(num2,den2,8)
```



(2) by Hand,

```
For step input, R(s)= 1/s, Y(s)= 1/{ s(s^2-3s+6) }

Take inverse Laplace Transform using,

Match: s^2-3s+6  to  s^2+2*ratio*wn*s + wn^2,
Result,
     wn=sqrt(6)=2.45 rad/sec
     ratio=-3/(2*wn) = -0.612
Then use,

               wn^2
     ------------------------   & its inverse,
     s(s^2+2*ratio*wn*s+wn^2)

1 – (1/sqrt(1-ratio^2))*exp(-ratio*wn*t)*sin(wn*sqrt(1-ratio^2)*t +
arcos(ratio))

Resulting to,

Step response of,
     y(t)=(1/6)*{ 1 – 1.26*exp(+1.5*t)*sin(1.94*t+arcos(-0.612) }
         = 0.167 – 0.21exp(1.5t)sin(1.94t+2.23)
```

Cleary, SYSTEM IS UNSTABLE.